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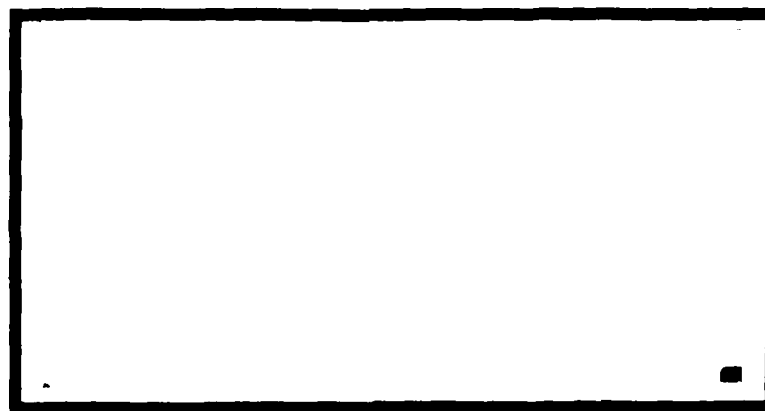
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OPTIMUM ORBIT PLANE CHANGE  
USING A SKIP REENTRY TRAJECTORY  
FOR THE SPACE SHUTTLE ORBITER

THESIS

AFIT/GA/AA/78D-4 ✓

Roger R. Harding  
2nd Lt USAF

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OPTIMUM ORBIT PLANE CHANGE  
USING A SKIP REENTRY TRAJECTORY  
FOR THE SPACE SHUTTLE ORBITER.

THESIS 9

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology  
Air University  
in Partial Fulfillment of the  
Requirements for the Degree of  
Master of Science

10  
by  
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2nd Lt USAF 12/66

Graduate Engineering Astrodynamics

11 December 1978

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## Preface

The multi-faceted mission of the Space Shuttle necessitates the orbiter's ability to change from one orbit to another. Many times an orbit change includes a change in orbit inclination which is usually accomplished by a rocket burn in space. The Space Shuttle Orbiter can possibly achieve changes in orbit inclination by making use of the orbiter's airplane-like ability in the upper atmosphere. This study examines this possibility and compares it with the rocket maneuver in space.

Most of the time spent on this study involved the optimization of such an aerodynamic maneuver. Unfortunately, no results from the optimization problem ensued; however, the problem is discussed and a better problem statement is proposed in Chapter VII.

Thanks are due to Captains William Wiesel and James Rader for their advice concerning the system dynamics and the optimum control problem. Captain Ray Barker was helpful in establishing a model atmosphere. Special thanks go to my wife, Wendy, for her expertise in editing and typing the final report.

Roger R. J. Harding

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Abstract

Two types of skip reentry trajectories are examined in the 70 km to 95 km altitude region. The first is a maximum lift-to-drag analysis which indicates that an aerodynamic maneuver in order to change the orbit inclination is profitable when compared to a rocket burn in space to effect the same change in orbit inclination. The maximum changes in orbit inclination achieved aerodynamically were approximately .8 degrees. The second type of analysis considered the optimal control problem for a skip reentry trajectory. The specific problem posed was: Find the optimum angle of attack and bank angle controls which minimize the amount of work done by drag for a specific change in orbit inclination. No results were obtained from this analysis due to the problems encountered when the optimization technique was applied to the specific problem.

# OPTIMUM ORBIT PLANE CHANGE USING A SKIP REENTRY TRAJECTORY FOR THE SPACE SHUTTLE ORBITER

## I Introduction

### Foreword

The Space Shuttle is a spacecraft system comprising three main elements: two solid propellant rocket boosters, an external tank, and the orbiter. The solid propellant rocket boosters are expended during the boost phase of the mission and are recovered from the ocean for reuse. The external tank remains coupled with the orbiter until after orbit insertion; when the fuel in the external tank is exhausted, the tank is jettisoned and reenters the earth's atmosphere. The orbiter is capable of delivering a variety of payloads into orbit as well as retrieving payloads for return to earth. The orbiter can also carry supplies and provide accommodations for up to four payload specialists as well as the crew of three (command pilot, copilot and mission specialist). Upon completion of orbital operations the orbiter reenters earth's atmosphere and terminates the mission with an airplane-like landing.

The purpose of this study is to examine the possibility of using the orbiter's aerodynamic capabilities to change the orbit inclination. Previously, spacecraft have changed orbit inclination by thrusting with rockets perpendicular to the orbital plane. Such a burn is expensive in terms of

fuel needed for the mission and therefore also expensive in terms of added payload weight during the boost phase.

The orbiter is unique in that it has a relatively good hypersonic lift-to-drag ratio (L/D) of approximately 2 (see Ref 8 ). A high L/D indicates a possibility that an aerodynamic maneuver during a skip reentry trajectory could provide an effective means for changing the orbit inclination. A skip reentry maneuver involves entering the earth's atmosphere, performing the desired aerodynamic maneuver and "skipping" back out of the atmosphere.

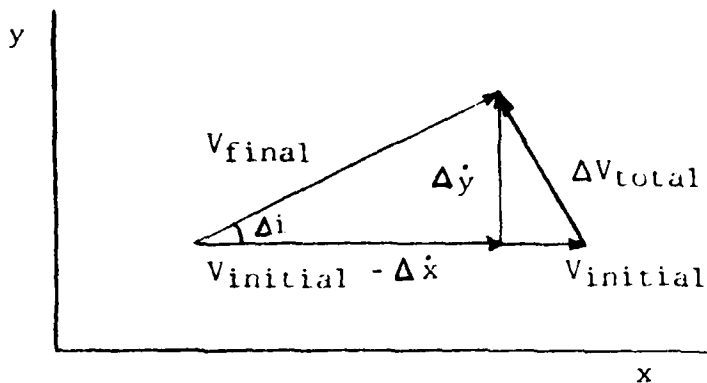
#### Back-of-the-Envelope Calculation

The relatively high L/D for the orbiter is the motivation for this study. A simplified calculation of an orbit plane change using L/D equal to 2 will follow; this calculation assumes all motion lies in a plane and the changes in velocity due to the aerodynamic forces are instantaneous. A typical reentry velocity is 7200 meters/second; a loss of 300 meters/second is assumed to be a maximum acceptable, since 300 meters/second is the velocity deficit which can be negated using the orbiter's maneuvering system (see Ref 8 ).

#### Example

$$V_{\text{initial}} = \dot{x} = 7200 \text{ m/s} ; \quad V_{\text{final}} = 7200 - 300 = 6900 \text{ m/s}$$

$$\dot{y}_{\text{initial}} = 0$$



Using Newton's Second Law, acceleration = ( $\Sigma$  Forces)/mass

$$\ddot{x} = \Delta \dot{x} = 300 \text{ m/s} = \text{drag/orbiter mass}$$

$$\ddot{y} = \Delta \dot{y} = \text{lift/orbiter mass}$$

Using these two equations,

$$\text{lift/drag} = 2 = \Delta \dot{y} / (300 \text{ m/s})$$

or

$$\Delta \dot{y} = 600 \text{ m/s}$$

From the geometry of the problem,

$$\text{total } \Delta V = (600^2 + 300^2)^{1/2} = 670 \text{ m/s}$$

so

$$i = \sin^{-1} \Delta \dot{y} / V_F$$

$$i \approx 5^\circ$$

The velocity of the orbiter lies in the orbit plane, thus the change in direction of the velocity vector,  $\Delta i$ , is also the change in the orbit plane. A plane change of approximately 5 degrees can be expected from such a skip turn maneuver according to this very rough analysis.

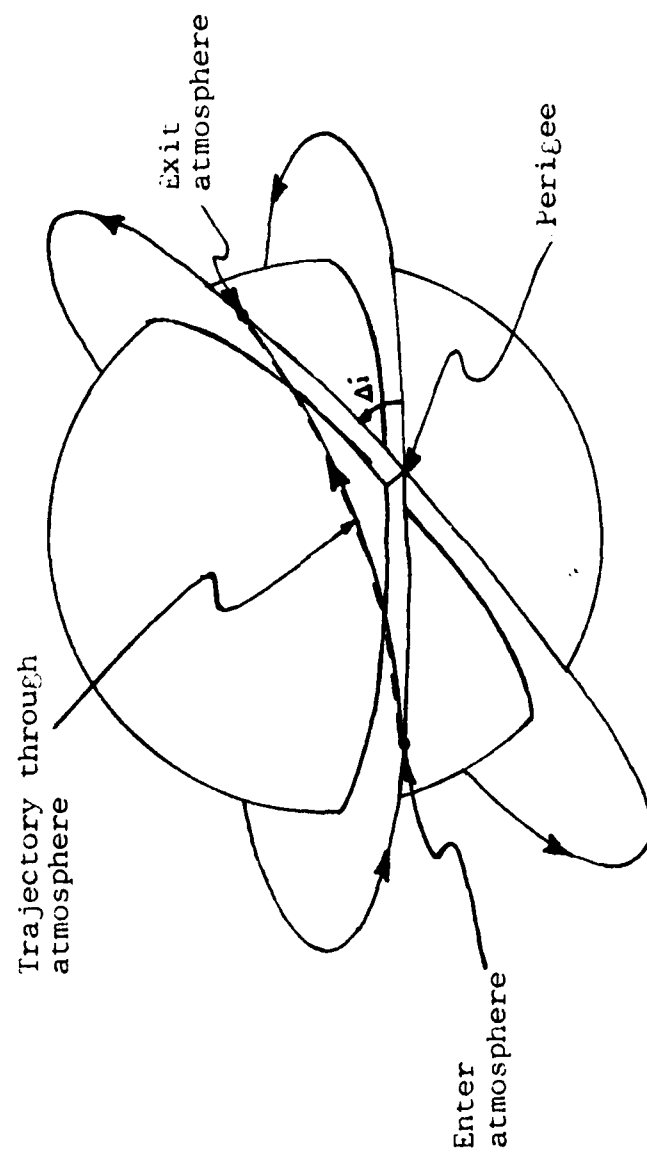


Figure I.1. Orbit plane change,  $\Delta i$

## Summary

This study examines a skip turn reentry maneuver in order to effect a change in orbit inclination for the Space Shuttle Orbiter.

Second-order parameter optimization finds optimum time histories for the controls, which in this case are angle of attack and bank angle. The optimization scheme finds these controls as polynomial functions of time while minimizing a performance index (work done by air drag). The problem is further constrained by specified final conditions, which in this case are maximum orbital plane change and attainment of a specific altitude upon completion of the maneuver. The theory for second-order parameter optimization, or suboptimal control as the procedure is often called, is presented in Chapter VI.

Before suboptimal trajectories can be determined, it is necessary to have a model atmosphere, the orbiter equations of motion, and the orbiter's aerodynamic data.

The conditions of the earth's upper atmosphere must be suitably modelled in order to evaluate the aerodynamic forces of lift and drag acting on the orbiter. Lift and drag are both functions of atmospheric density, temperature, kinetic viscosity and molecular weight; the 1962 Standard Atmosphere provides reasonable approximations of same. The orbiter aerodynamic data obtained from NASA is based on the '62 atmosphere; hence the 1962 Standard Atmosphere is the model used in this study.

Solution of the optimal control problem requires the integration of the system equations of motion from a prescribed initial state to a final state. The equations of motion are derived by transforming the forces acting on the orbiter from a body-fixed reference frame to the geocentric, inertial reference frame and then applying Newton's Second Law. The forces acting on the orbiter are gravity, lift, and drag. The equations of motion are thus dependent on the model atmosphere.

The orbiter aerodynamic data necessary for this study are the lift and drag coefficients. The coefficients are dependent upon the atmospheric properties, the orbiter's aerodynamic configuration (angle of attack), and the state of the earth-orbiter system.

These three prerequisites necessary for the optimization scheme are quite different when taken individually but become interdependent when applied to this problem. The equations of motion are dependent on the aerodynamic forces which in turn are dependent on the orbiter's aerodynamic coefficients and the model atmosphere. The orbiter aerodynamic coefficients are also dependent on the model atmosphere and the state of the system. The state of the system is evaluated by integrating the equations of motion. The integration is begun by choosing an initial state and allowing a numerical integration scheme to propagate this initial state forward in time.

Suboptimal control requires integrating the equations of motion using guessed polynomials for the controls. The

performance index and final state are recorded. The coefficients in the control polynomials are changed slightly and the equations of motion are integrated again using the new control polynomials. Central differencing uses changes in both performance index and final end conditions with respect to the change in polynomial coefficients in order to evaluate the first and second partial derivatives. These partial derivatives are used in suboptimal control theory to make a better guess at the coefficients in the control polynomials. The process is repeated until changes in the control polynomial coefficients become very small.

For a prescribed final altitude and orbit plane change, optimum control polynomials are found which yield the minimum work done by drag on the orbiter.

## II Model Atmosphere

### Background

The aerodynamic forces acting on the Space Shuttle Orbiter are a function of the state of the system, i.e. altitude and velocity. Knowledge of atmospheric properties at low density altitudes is essential for hypersonic aerodynamic trajectory calculations. The 1962 Standard Atmosphere used by this study provides the necessary atmosphere properties. The aerodynamic forces are evaluated by solving the following equations.

$$\text{Lift} = \frac{1}{2} V \rho C_L S$$

$$\text{Drag} = \frac{1}{2} V \rho C_D S$$

where

$\rho$  = local atmospheric density

$V$  = free stream velocity relative to the orbiter

$C_L$  = lift coefficient

$C_D$  = drag coefficient

$S$  = reference area

Lift and drag are dependent on atmospheric properties in two ways: (1) directly dependent on air density, and (2) indirectly dependent on the temperature, molecular weight, density and kinetic viscosity used for evaluating lift and drag coefficients. Thus an accurate model atmosphere is essential in order to evaluate the lift and drag forces and the resulting trajectory.

### The 1962 Standard Atmosphere

The 1962 Standard Atmosphere model is an updated version of earlier atmospheres taking advantage of increased knowledge and more accurate determinations of basic quantities. This model extends to 700 km altitude and assumes an ideal gas devoid of moisture, water vapor, dust and obeys the perfect gas law. Further, the '62 model is idealized to a mid-latitude year-round mean over the range of maximum and minimum solar activity (Ref 9). Temperature variations and molecular weight variations with altitude were obtained experimentally.

The 1962 Standard Atmosphere is used in this study for two reasons: the Space Shuttle Orbiter aerodynamic data is based on the '62 atmosphere, and this model atmosphere is accurate and easy to implement in this study.

### Defining Equations

Atmospheric parameters of interest in this study are temperature, molecular weight of air, density and viscosity. Upper atmosphere winds are also of interest; however, due to the complexity of the winds (variations with time and position) the model simplifies the winds to an atmosphere rotating with the earth.

The variation of temperature,  $T$ , with altitude is taken to be a piecewise continuous function with constant gradients,  $L_m$ . This model atmosphere uses molecular temperature,  $T_m$ , as the defining property:

$$T_m = T\left(\frac{M_0}{M}\right)$$

Molecular weight of air,  $M$ , is a constant,  $M_0$ , up to 90 km

altitude. For this altitude regime, the molecular temperature and the temperature are equal. The interpolating function for the molecular temperature is:

$$T_m = T_{mb} + L_m (Z - Z_b)$$

The subscript b denotes a base altitude which is the altitude where the local temperature gradient changes.

Values for the molecular weight of air are available from the model atmosphere only at the base altitudes. However, the molecular temperature accounts for the varying molecular weight. All parameters involving molecular weight can be expressed in terms of temperature and molecular temperature.

The derivation of the density equation uses the perfect gas law, the standard barometric equation, and geopotential altitude. Due to the variation of gravitational acceleration with altitude, geopotential altitude represents a modified altitude to account for this variation. This study assumes that the geopotential altitude is equal to the geometric altitude. This simplifying assumption results in a negligible five to seven percent difference from the 1962 Standard Atmosphere at high altitudes.

Two equations for density,  $\rho$ , are derived:

$$\frac{\rho}{\rho_0} = \frac{\rho_b}{\rho_0} \exp - \frac{Q (Z - Z_b)}{T_{mb}} \quad \text{for } L_m = 0$$

and

$$\frac{\rho}{\rho_0} = \frac{\rho_b}{\rho_0} \exp \left( 1 + \frac{Q}{L_m} \right) \left( \log^{-1} \left( \frac{T_{mb}}{T_m} \right) \right) \quad \text{for } L_m \neq 0$$

where

$\rho_0$  = atmospheric density at sea level

$\rho_b/\rho_0$  = known ratio at the base altitudes

$$Q = (\mu_0 M) / (r^2 R)$$

The scale height,  $Q$ , is dependent on the local gravitational acceleration,  $\mu_0/r^2$ , the molecular weight of air,  $M$ , and the universal gas constant,  $R$ .

Viscosity is a necessary parameter for the calculation of the viscous parameter,  $VBAR$ . Viscosity is assumed to exist at all altitudes and is determined as a function of temperature,  $T$ :

$$\mu = \frac{T^{3/2}}{(T + S)} \beta$$

where

$$S = 110.4 \text{ K}$$

$$\beta = 1.458E+06 \text{ kg/mK}^{1/2}\text{s}$$

At any given altitude between sea level and 700 km temperature, molecular weight, density and viscosity can be determined using the 1962 Standard Atmosphere. These parameters are used to evaluate the aerodynamic forces and to integrate the system equations of motion.

### III System Dynamics

Evaluating Space Shuttle Orbiter trajectories and orbit plane changes involves integrating the system's equations of motion from a given initial state to a final state. The equations of motion are derived by analyzing system dynamics and applying Newton's Second Law.

#### Assumptions

The derivation of the equations of motion will involve the following assumptions:

1. the earth is spherical
2. the earth is inertial
3. the orbiter is a point mass with lift and drag acting through the center of gravity
4. no sideslip occurs
5. atmospheric wind is taken to be the rotational velocity of the earth at orbiter's altitude.

These assumptions greatly simplify the dynamics without detracting substantially from the credibility of the study. The assumptions that the earth is spherical and inertial are reasonable approximations since the skip reentry trajectory maneuver takes a relatively short time; the time span for such a maneuver is 27 minutes or less. This time span is not long enough for the effects of an actual non-inertial, non-spherical earth to produce an appreciable difference from the simplified two-body case being studied here.

The assumption that the orbiter is a point mass with the

aerodynamic forces acting through the center of mass is realistic in that the orbiter would be trimmed (i.e. no pitching, yawing or rolling moments) at all times during a skip reentry maneuver.

It is also realistic to assume that no sideslip occurs. The hypersonic velocities severely limit the configurations of an orbital vehicle reentering the atmosphere. This problem is due to extremely high aerodynamic heating faced by any reentry vehicle. Aerodynamic heating is highest on surfaces of the vehicle exposed to the freestream flow. The orbiter's primary heat-protective surfaces are around the nose and along the underside of the vehicle with secondary heat-protective surfaces along the sides and top. Thus the relative velocity vector must lie along or as close as possible to the longitudinal center line of the orbiter.

Trying to accurately model the upper atmospheric winds would be extremely complex. Such a wind model would involve time of day, seasons, position, etc. Incorporating such a model goes beyond the scope of this study; however, assuming an inertial atmosphere is probably not justifiable. Assuming an atmosphere rotating with the earth is a sufficient wind model for this study.

#### Equations of Motion

The equations of motion are derived by establishing the appropriate coordinate systems and applying Newton's Second law. Two coordinate frames are used. The  $\bar{X}\bar{Y}\bar{Z}$  frame is inertial and fixed at the earth's center. The  $\bar{V}_{rel}\bar{M}\bar{L}$  frame is fixed at the orbiter's center of mass (Fig III.1).

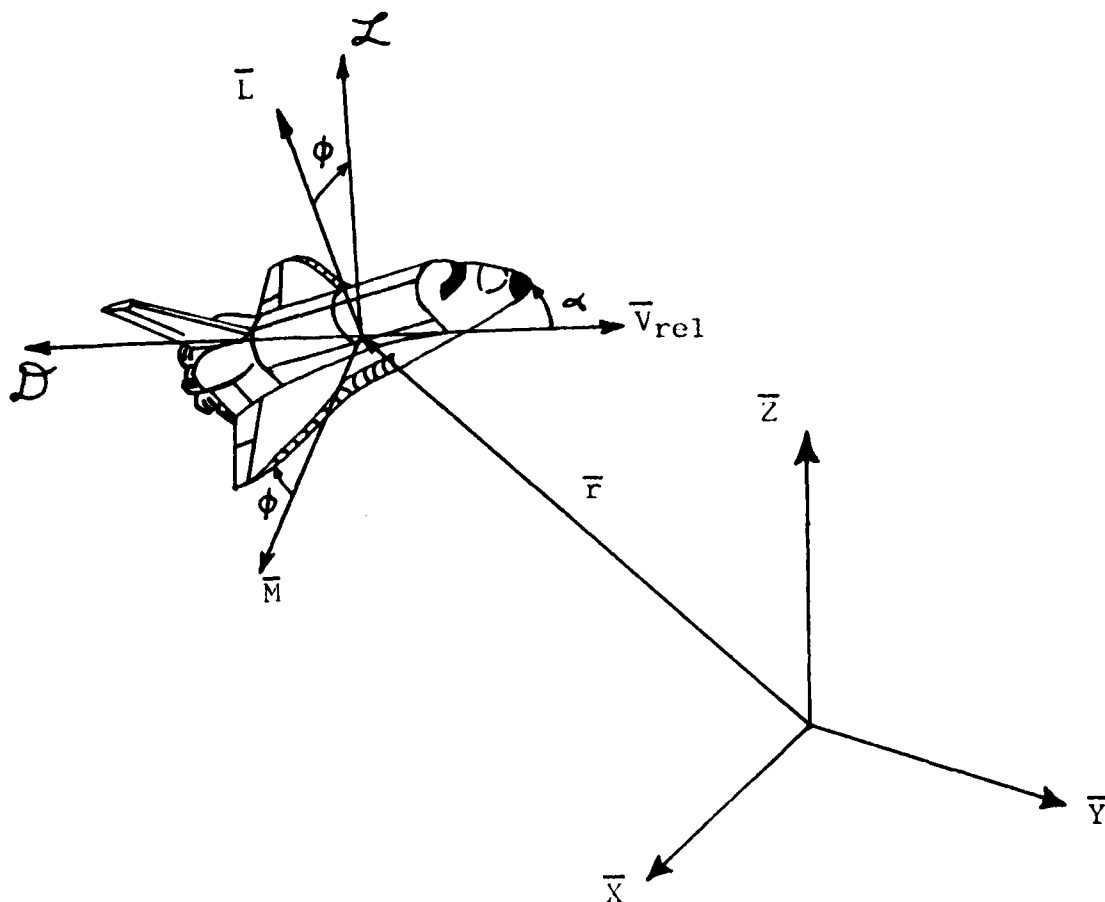


Figure III.1. Earth-orbiter system coordinate frames

The state of the system can be described by the state variables  $x, y, z$ , and  $\dot{x}, \dot{y}, \dot{z}$ . These quantities are evaluated with respect to the inertial frame.

The  $\bar{V}_{rel} \bar{M} \bar{L}$  frame is defined in terms of the state variables.

$$\bar{V}_{rel} = \bar{V} + \bar{\omega} \times \bar{r}$$

$$\bar{M} = \bar{V}_{rel} \times \bar{r}$$

$$\bar{L} = \bar{M} \times \bar{V}_{rel}$$

where

$$\bar{V} = x\hat{i} + y\hat{j} + z\hat{k} \quad (\text{m/s})$$

$$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad (\text{m})$$

$$\bar{\omega} = 7.292115856 \times 10^{-5} \hat{k} \quad (\text{radians/sec})$$

Unit vectors are denoted by the hat symbol,  $\hat{\phantom{x}}$ , and  $\hat{i}, \hat{j}, \hat{k}$  represent unit vectors for the  $\bar{X}\bar{Y}\bar{Z}$  frame. The angular velocity of the earth is constant and denoted by  $\bar{\omega}$ . Thus  $\bar{V}_{rel}$  is the velocity of the orbiter relative to the atmosphere,  $\bar{M}$  establishes a local horizontal and  $\bar{L}$  completes the right-hand coordinate frame.

Aerodynamic forces are easily expressed in the body-centered frame (Fig III.1).

$$\overline{\text{Lift}} = \mathcal{L}(\cos \phi \hat{L} - \sin \phi \hat{M})$$

$$\overline{\text{Drag}} = -\mathcal{D} \hat{V}_{rel}$$

where

$\mathcal{L}$  = magnitude of the lift vector

$\mathcal{D}$  = magnitude of the drag vector

$\phi$  = bank angle of the orbiter

The unit vectors can be expressed in the  $\overline{XYZ}$  coordinate frame. A transformation matrix is needed to express the forces acting on the orbiter in terms of the inertial frame. The unit vectors,  $\hat{V}_{rel} \hat{M} \hat{L}$ , provide a means for transforming an aerodynamic vector in the body frame to the inertial frame because the unit vectors are expressed in the inertial frame. Thus a transformation matrix,  $T$ , can be formed by using the unit vectors as column vectors.

$$T = \begin{vmatrix} \hat{V}_{rel} & \hat{M} & \hat{L} \end{vmatrix}$$

Expanding the defining equations for the  $V_{rel} ML$  coordinate frame starts the derivation for the transformation matrix.

$$\begin{aligned} \overline{V}_{rel} &= (x - \omega y)\hat{i} + (y + \omega x)\hat{j} + z\hat{k} \\ &= a_v\hat{i} + b_v\hat{j} + c_v\hat{k} \\ \overline{M} &= \overline{V}_{rel} \times \overline{r} = (b_v z - c_v y)\hat{i} - (a_v z - c_v x)\hat{j} + (a_v y - b_v x)\hat{k} \\ &= a_m\hat{i} + b_m\hat{j} + c_m\hat{k} \\ \overline{L} &= \overline{M} \times \overline{V}_{rel} = (c_v b_m - b_v c_m)\hat{i} - (c_v a_m - a_v c_m)\hat{j} \\ &\quad + (b_v a_m - a_v b_m)\hat{k} \\ &= a_1\hat{i} + b_1\hat{j} + c_1\hat{k} \end{aligned}$$

Thus the transformation matrix becomes

$$T = \begin{vmatrix} a_v/s_v & a_m/s_m & a_1/s_1 \\ b_v/s_v & b_m/s_m & b_1/s_1 \\ c_v/s_v & c_m/s_m & c_1/s_1 \end{vmatrix}$$

where

$$S_i = (a_i^2 + b_i^2 + c_i^2)^{1/2} \quad i = v, m, 1$$

The equations of motion can be reckoned using Newton's Second Law.

$$\Sigma \vec{F} = m\vec{a}$$

where

$$\begin{aligned}\Sigma \vec{F} &= (\vec{F}_{\text{gravity}})_{XYZ} + (\vec{\text{Lift}})_{XYZ} + (\vec{\text{Drag}})_{XYZ} \\ &= (\vec{F}_{\text{gravity}})_{XYZ} + T(\vec{\text{Lift}})_{VML} + T(\vec{\text{Drag}})_{VML}\end{aligned}$$

Further,

$$(\vec{F}_{\text{gravity}})_{XYZ} = - \frac{GMm}{r^3} (x\hat{i} + y\hat{j} + z\hat{k})$$

where

$GM$  = earth's gravitational parameter

$m$  = mass of the orbiter

$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

$$(\text{lift})_{VML} = \begin{vmatrix} 0 \\ -L \sin \phi \\ L \cos \phi \end{vmatrix}$$

$$(\text{Drag})_{VML} = \begin{vmatrix} -D \\ 0 \\ 0 \end{vmatrix}$$

By substituting into Newton's Second Law the equations of motion can be found. In component form the equations are:

$$\ddot{x} = - (GM/r^3)x + \frac{L}{m} (\cos \phi \frac{a_1}{s_1} - \sin \phi \frac{a_m}{s_m}) - \frac{D}{m} \frac{a_v}{s_v}$$

$$\ddot{y} = - (GM/r^3)y + \frac{L}{m} (\cos \phi \frac{b_1}{s_1} - \sin \phi \frac{b_m}{s_m}) - \frac{D}{m} \frac{b_v}{s_v}$$

$$\ddot{z} = - (GM/r^3)z + \frac{L}{m} (\cos \phi \frac{c_1}{s_1} - \sin \phi \frac{c_m}{s_m}) - \frac{D}{m} \frac{c_v}{s_v}$$

These three second-order equations can be broken down to six first-order equations by redefining the state variables.

$$w_1 = x$$

$$w_2 = y$$

$$w_3 = z$$

$$w_4 = \dot{x}$$

$$w_5 = \dot{y}$$

$$w_6 = \dot{z}$$

The equations of motion become

$$\dot{w}_1 = w_4$$

$$\dot{w}_2 = w_5$$

$$\dot{w}_3 = w_6$$

$$\dot{w}_4 = - (GM/r^3)w_1 + \frac{x}{m} \left( \cos \frac{a_1}{s_1} - \sin \frac{a_m}{s_m} \right) - \frac{D}{m} \frac{a_v}{s_v}$$

$$\dot{w}_5 = - (GM/r^3)w_2 + \frac{x}{m} \left( \cos \frac{b_1}{s_1} - \sin \frac{b_m}{s_m} \right) - \frac{D}{m} \frac{b_v}{s_v}$$

$$\dot{w}_6 = - (GM/r^3)w_3 + \frac{x}{m} \left( \cos \frac{c_1}{s_1} - \sin \frac{c_m}{s_m} \right) - \frac{D}{m} \frac{c_v}{s_v}$$

The a's, b's c's and s's are also functions of the new variables. These six first-order equations are integrated through time by the CC6600 differential equation integrator, CDE.

#### Orbit Plane Change Equation

The change in orbit inclination, or orbit plane change, due to the skip reentry maneuver is determined by comparing the states of the system before and after the maneuver. The angular momentum vector,  $\bar{H}$ , involves the complete state of the

orbiter and it is also normal to the orbit plane. Thus the orbit plane change can be found by examining how the angular momentum vector changes direction (Fig III.2). The inner product for two vectors is well-suited for this problem. Let the subscripts i and f denote initial and final states respectively.

$$\bar{H}_i \cdot \bar{H}_f = |\bar{H}_i| |\bar{H}_f| \cos \Delta i$$

$\bar{H}_i$  and  $\bar{H}_f$  indicate the magnitudes of the initial and final angular momentum. Further,

$$\bar{H} = \bar{r} \times \bar{v}$$

$\Delta i$  = orbit plane change

Rearranging:

$$\Delta i = \cos^{-1} \frac{(\bar{r}_i \times \bar{v}_i) \cdot (\bar{r}_f \times \bar{v}_f)}{|\bar{r}_i \times \bar{v}_i| |\bar{r}_f \times \bar{v}_f|}$$

The plane change calculation is easily carried out using the above equation.

Integrating the equations of motion to a desired time and evaluating the change in the state of the system at this time represent the fundamental dynamics of the problem. Evaluations of the aerodynamic forces and the performance index are discussed in the following chapters.

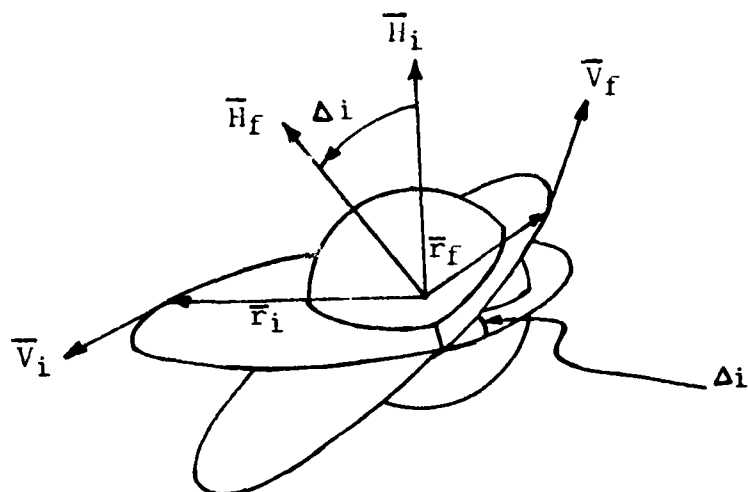


Figure III.2. Initial and final angular momentum vectors

#### IV Space Shuttle Orbiter Aerodynamics

The orbiter is assumed to be a point mass with the aerodynamic forces, lift and drag, acting through the center of mass. This assumption simplifies the orbiter aerodynamics to evaluating only lift and drag.

Lift and drag forces are dependent upon the state of the system; thus the forces must be constantly evaluated during the integration of the equations of motion.

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \rho(x,y,z) C_L(x,y,z,x,y,z) V_{rel}^2(x,y,z,x,y,z) \\ \mathcal{D} &= \frac{1}{2} \rho(x,y,z) C_D(x,y,z,x,y,z) V_{rel}^2(x,y,z,x,y,z) \end{aligned}$$

The atmospheric density,  $\rho$ , is obtained from the model atmosphere and  $V_{rel}$  is obtained continuously during the integration of the equations of motion. This chapter explains how the lift and drag coefficients,  $C_L$  and  $C_D$ , are determined.

##### Orbiter Aerodynamic Data

The orbiter aerodynamic data was obtained from the Space Shuttle reentry office of NASA in Houston, Texas. The information included lift and drag coefficient tables as functions of angle of attack,  $\alpha$ , and the viscous parameter,  $VBAR$  (see Table IV.1). Also included in the data package was the information necessary to evaluate  $VBAR$  and an explanation of how the lift and drag coefficients for the orbiter were derived.

The viscous parameter is a subscale-fullscale simulation parameter based on the freestream properties Mach number,  $M_\infty$ , Reynolds number,  $Re$ , and Temperature,  $T$ .  $VBAR$  is defined as

$$V_{BAR} = M_{\infty} (C_{\infty}/Re)^{\frac{1}{2}} \quad (\text{Ref 4})$$

where

$$C_{\infty} = (\Gamma'/T)^k \frac{T + 122.1 \times 10^{-(5/T)}}{T + 122.1 \times 10^{-(5/T')}}$$

with

$$T' = T (.468 + .532(T_w/T) + .195(\gamma-1)/2 M^2)$$

$$k = .5$$

$$T_w = 2000 \text{ F}$$

$$\gamma = 1.15$$

C is the proportionality factor for the linear viscosity-temperature relationship. K is an empirical constant. The specific heat ratio,  $\gamma$ , is assumed to be constant for the flight conditions analyzed. A constant wall temperature,  $T_w$ , is also assumed for the orbiter.

The freestream Mach number is the ratio of  $V_{rel}$  to the freestream speed of sound,  $a_{\infty}$ . The speed of sound is assumed to exist at all altitudes and can be evaluated using the following equation:

$$a_{\infty} = (R T_M / M_0)^{\frac{1}{2}}$$

where

$$R = 8.31432 \times 10^3 \text{ N-m/kg K}$$

The freestream Reynolds number is based on the orbiter length,  $s = 32.77 \text{ m}$ .

$$Re = \rho V_{rel} s / \mu$$

Kinetic viscosity,  $\mu$ , is assumed to exist at all altitudes and is obtained from the model atmosphere.

For values of  $V_{BAR}$  greater than .01 and less than .08 the lift and drag coefficients have been determined experimentally. For values of  $V_{BAR}$  greater than .08 to its maximum value of 5.2, the coefficients are determined analytically using the Lockheed Empirical Bridging Formula (Appendix A). This formula bridges the transitional flow regime between continuum and free-molecular flow regimes.

These tables of aerodynamic coefficients are based on an assumed nominal reentry trajectory and therefore nominal velocities. Thus,  $V_{BAR}$  is a parameter dependent primarily on temperature (i.e. altitude). Typically, the extreme values of  $V_{BAR}$ , .01 and 5.2, correspond to altitudes of 67 km and 190 km respectively.

Given the state of the system,  $V_{BAR}$  can be evaluated. With  $V_{BAR}$  and the angle of attack available, the corresponding aerodynamic coefficient can be found. The data is tabulated at discrete values of angle of attack and  $V_{BAR}$ . A four point bivariate interpolation scheme is employed to evaluate the coefficients for angles of attack and  $V_{BAR}$ 's between the discrete data points. The bivariate interpolation scheme is a second-order approximation (Appendix B).

VBAR =	<u>.010</u>	<u>.020</u>	<u>.040</u>	<u>.060</u>	<u>.080</u>	<u>.310</u>
ALPHA						
20.0	.190800	.207570	.232000	.256430	.280870	.477820
25.0	.299500	.316860	.341420	.366070	.390630	.606860
30.0	.446700	.458290	.481760	.505320	.528790	.758220
35.0	.624700	.631590	.652640	.673690	.694740	.930470
40.0	.834100	.842920	.861230	.879460	.897770	1.130280
45.0	1.066900	1.075010	1.090360	1.105700	1.121050	1.346820
50.0	1.327700	1.334140	1.346280	1.358370	1.370520	1.580650

Table IV.1. Orbiter Drag Coefficients

VBAR =	<u>.920</u>	<u>1.650</u>	<u>2.500</u>	<u>5.050</u>	<u>5.200</u>
ALPHA					
20.0	.882830	1.157240	1.273170	1.372190	1.393070
25.0	1.052340	1.344690	1.480870	1.588750	1.606330
30.0	1.230150	1.545790	1.682720	1.800290	1.827540
35.0	1.415310	1.704850	1.878780	2.001070	2.029030
40.0	1.608470	1.891120	2.062830	2.186220	2.223940
45.0	1.811110	2.083910	2.248670	2.372060	2.406000
50.0	2.012720	2.267240	2.415300	2.534810	2.571760

Table IV.1. Orbiter Drag Coefficients - Concluded

VBAR =	<u>.010</u>	<u>.020</u>	<u>.040</u>	<u>.060</u>	<u>.080</u>	<u>.310</u>
ALPHA						
20.0	.337570	.330540	.321650	.312760	.303860	.261980
25.0	.485270	.483820	.472370	.460870	.449420	.383570
30.0	.636890	.627990	.614440	.600840	.587290	.497080
35.0	.770820	.758510	.743770	.729030	.714290	.600260
40.0	.884150	.877600	.862240	.846940	.831580	.693920
45.0	.967350	.960750	.945400	.930060	.914710	.762330
50.0	1.021660	1.015560	1.001080	.986680	.972200	.807810

Table IV.2. Orbiter Lift Coefficients

VBAR =	.920	1.650	2.500	5.050	5.200
ALPHA					
20.0	.175970	.109500	.078910	.072030	.087310
25.0	.249980	.150510	.106540	.084480	.104420
30.0	.311680	.176790	.117360	.087590	.109390
35.0	.365520	.219910	.123270	.082080	.101440
40.0	.410810	.237990	.120280	.068830	.096830
45.0	.448940	.258310	.127920	.070290	.089940
50.0	.469620	.262590	.123790	.061020	.080150

Table IV.2. Orbiter Lift Coefficients - Concluded

## V Maximum Lift-to-Drag Trajectories

Analyzing maximum lift-to-drag L/D trajectories serves two purposes: first, such an analysis will indicate whether or not a skip turn reentry trajectory is profitable when compared to a rocket maneuver outside the atmosphere; and, second, the analysis will also indicate how large an orbit plane change can be expected.

It should be noted here that this analysis is different from the optimal control method explained in the next chapter in that energy lost to air drag is not minimized, nor is the change in orbit inclination maximized. Maximum L/D analyses are indicative of the performance of any aerodynamic vehicle.

A maximum L/D trajectory implies that at each point along the aerodynamic maneuver, the orbiter is oriented to the angle of attack corresponding to the maximum L/D ratio.

### Velocity Performance Index

In order to effect an orbit plane change in space, a rocket burn directed outside of the orbit plane is necessary. Thus, given an initial orbit and a desired plane change, a corresponding change in velocity,  $\Delta V$ , can be calculated. The equation for this computation is:

$$\Delta V = 2V_i \sin (\Delta i/2)$$

where

$V_i$  = velocity prior to the maneuver

$\Delta i$  = orbit plane change

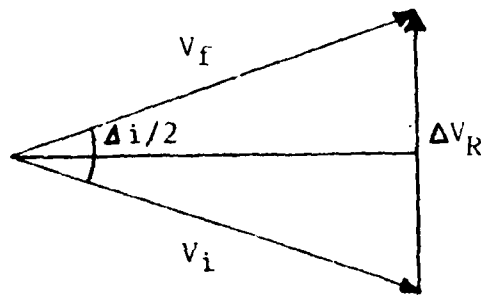


Figure V.1 Velocity Vector Geometry

This equation assumes circular initial and final orbits with equal initial and final velocities.

Using a skip turn reentry maneuver to effect an orbit plane change results in energy lost due to air drag. Because the maneuver starts and ends at the same altitude, the energy lost is all kinetic energy, i.e. the maneuver results in a  $\Delta V$ . The question of whether or not the aerodynamic maneuver is profitable can now be properly posed: Given an orbit plane change, is the  $\Delta V$  required for a rocket maneuver,  $\Delta V_R$ , greater than the  $\Delta V$  required for an aerodynamic maneuver,  $\Delta V_D$ ? Or asking the same question differently: For a given orbit plane change, is the ratio  $\Delta V_R/\Delta V_D$  greater than 1? If the answer to these questions is yes, then an aerodynamic maneuver is profitable.

The ratio  $\Delta V_R/\Delta V_D$  is called the velocity performance index. The maximum L/D analysis compares the velocity performance indices for bank angles varying from 0 to 90 degrees at perigee altitudes of 95, 90, 85, 80, 75 and 70 km. These perigee altitudes correspond to initial conditions neglecting

the atmosphere. Actual perigee altitudes will vary from those listed above due to atmospheric effects.

#### Discussion of Results

Two different types of graphs are shown in the following figures: velocity performance index vs. bank angle, and orbit inclination change vs. bank angle.

The velocity performance index for perigee altitudes less than and equal to 85 km exceeds one, as shown in Figures V.4 through V.7. The bank angle at which the velocity performance index becomes greater than one represents the minimum bank angle for the corresponding perigee altitude at which the aerodynamic maneuver becomes profitable over the rocket burn maneuver in space.

For a perigee altitude of 75 km, the graph ends at 70 degrees bank angle (Fig. V.6). If the bank angle exceeds 70 degrees, the aerodynamic maneuver is no longer a skip maneuver; it becomes a total reentry where the trajectory does not leave the atmosphere. A similar situation occurs for bank angles greater than 50 degrees for a perigee altitude equal to 70 km. Altitudes less than 70 km are not within the altitude regime for which orbiter aerodynamic data is available.

Changes in orbit inclination are insignificant for perigee altitudes greater than 80 km (Fig. V.8 through V.13). Maximum obtainable orbit inclination changes for perigee altitudes less than 80 km are approximately .8 degrees. Velocity lost due to air drag is another parameter of concern. Typically, velocity lost to drag while obtaining .8 degrees orbit

inclination change ranges from 60 m/s to 80 m/s. If 300 m/s is an acceptable velocity loss, then orbit inclination changes greater than 1 degree are obtainable.. This can be accomplished by orienting the initial orbit such that the orbiter enters the atmosphere earlier and leaves later. An alternative would be to perform the desired orbit inclination change over a series of skip turn reentry trajectories, i.e. a multiple orbit revolution maneuver.

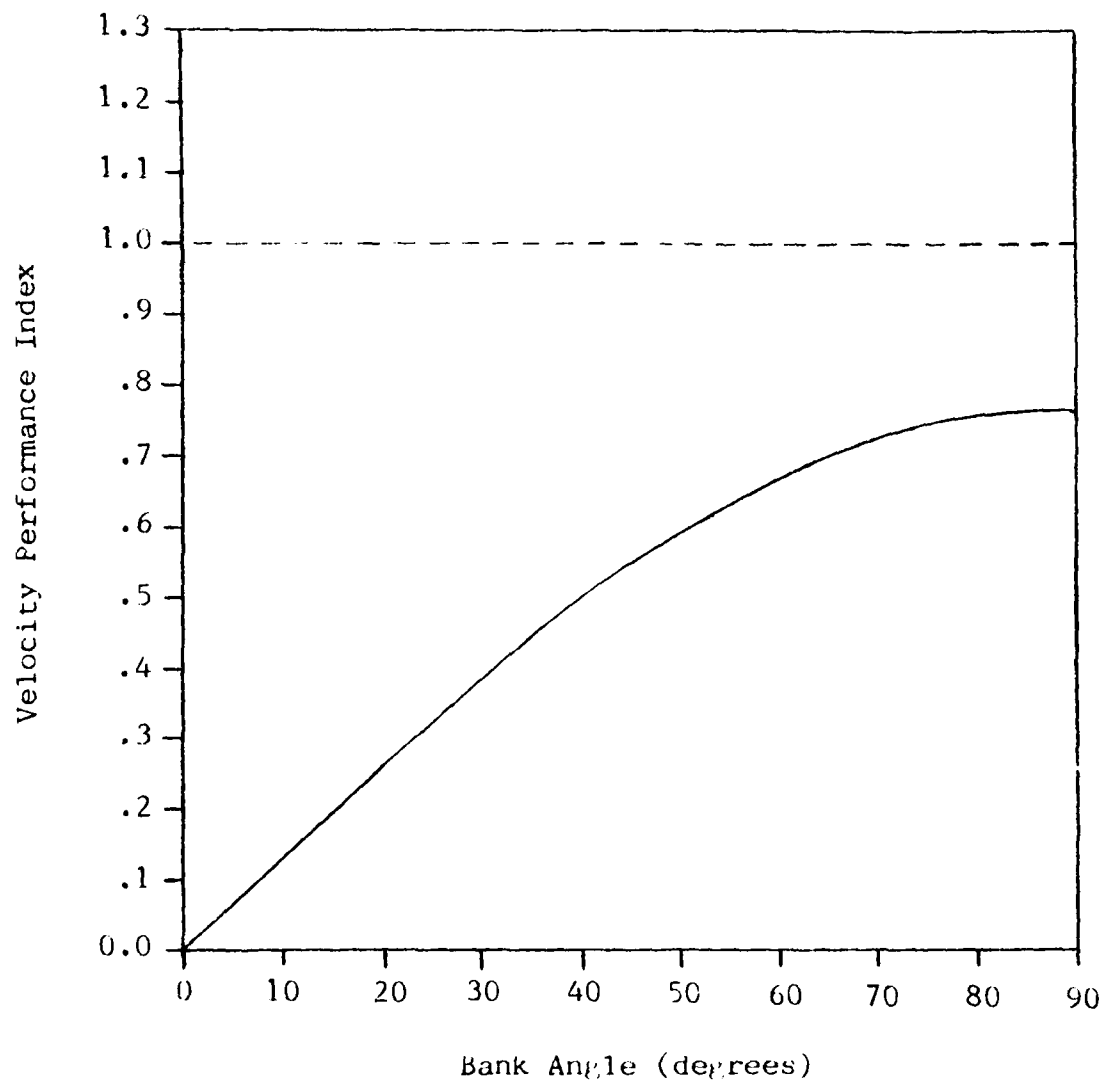


Figure V.2. Velocity Performance Index for 95 km Perigee Altitude

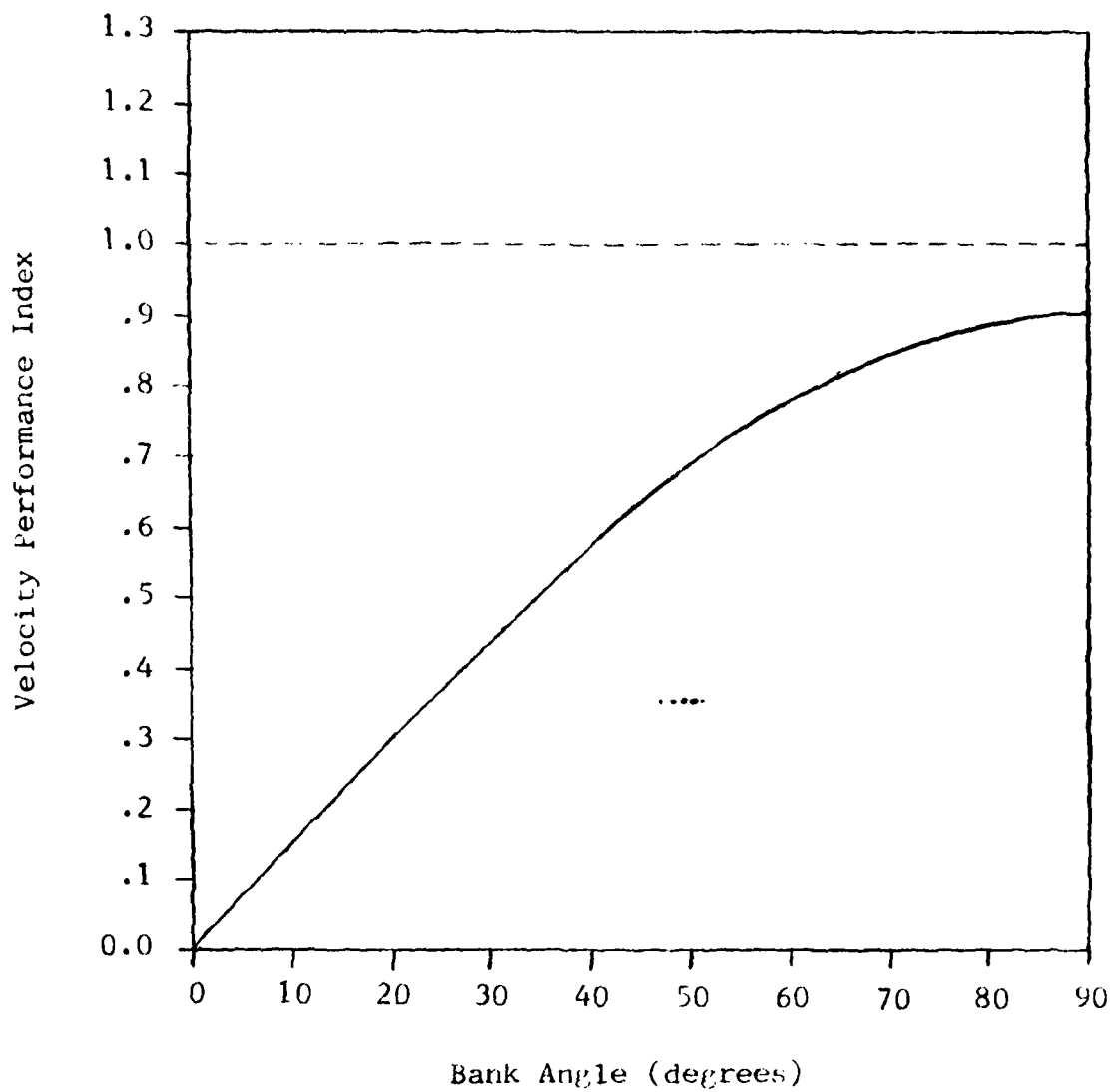


Figure V.3. Velocity Performance Index for 90 km Perigee Altitude

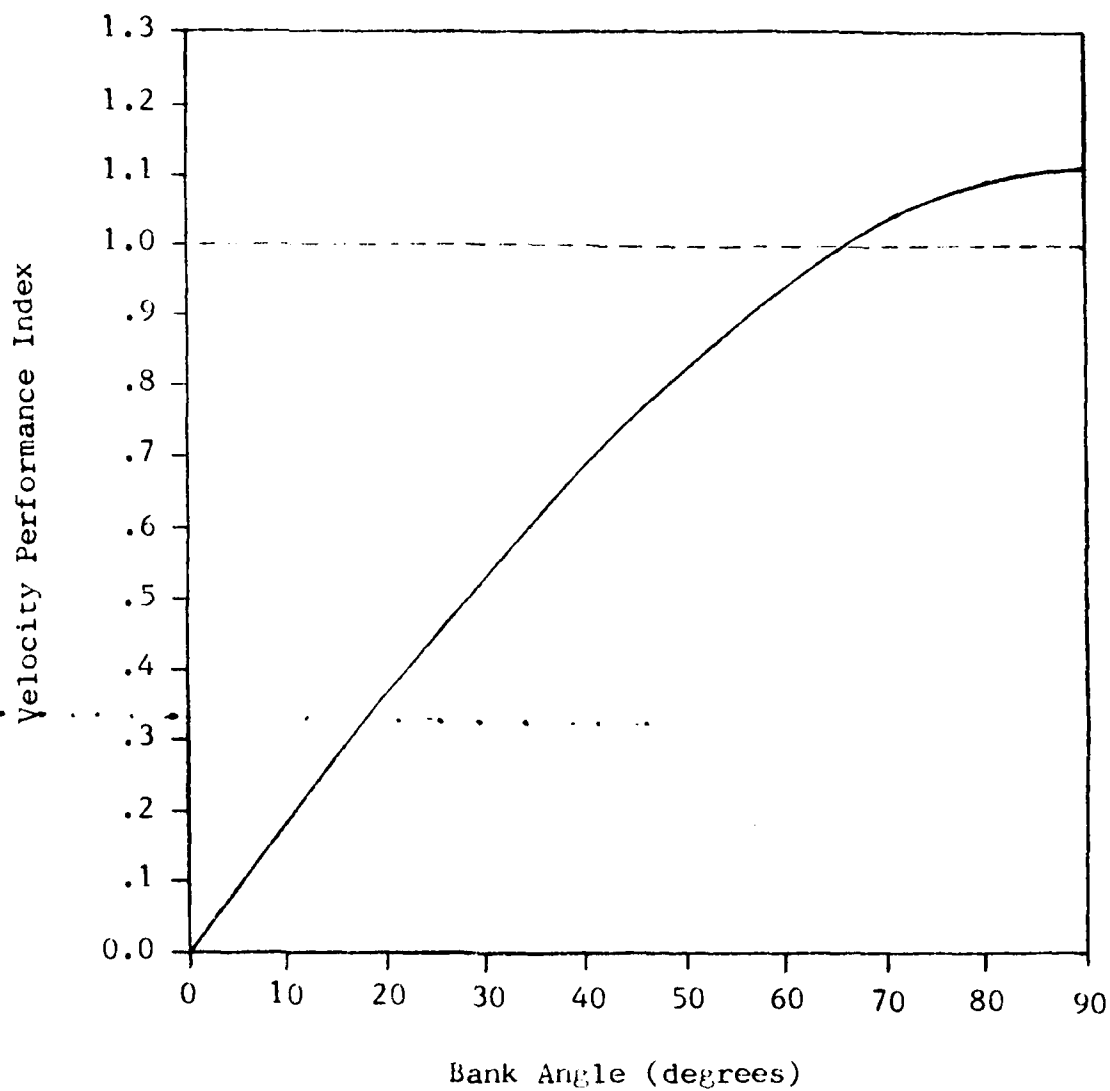


Figure V.4. Velocity Performance Index for 85 km Perigee Altitude

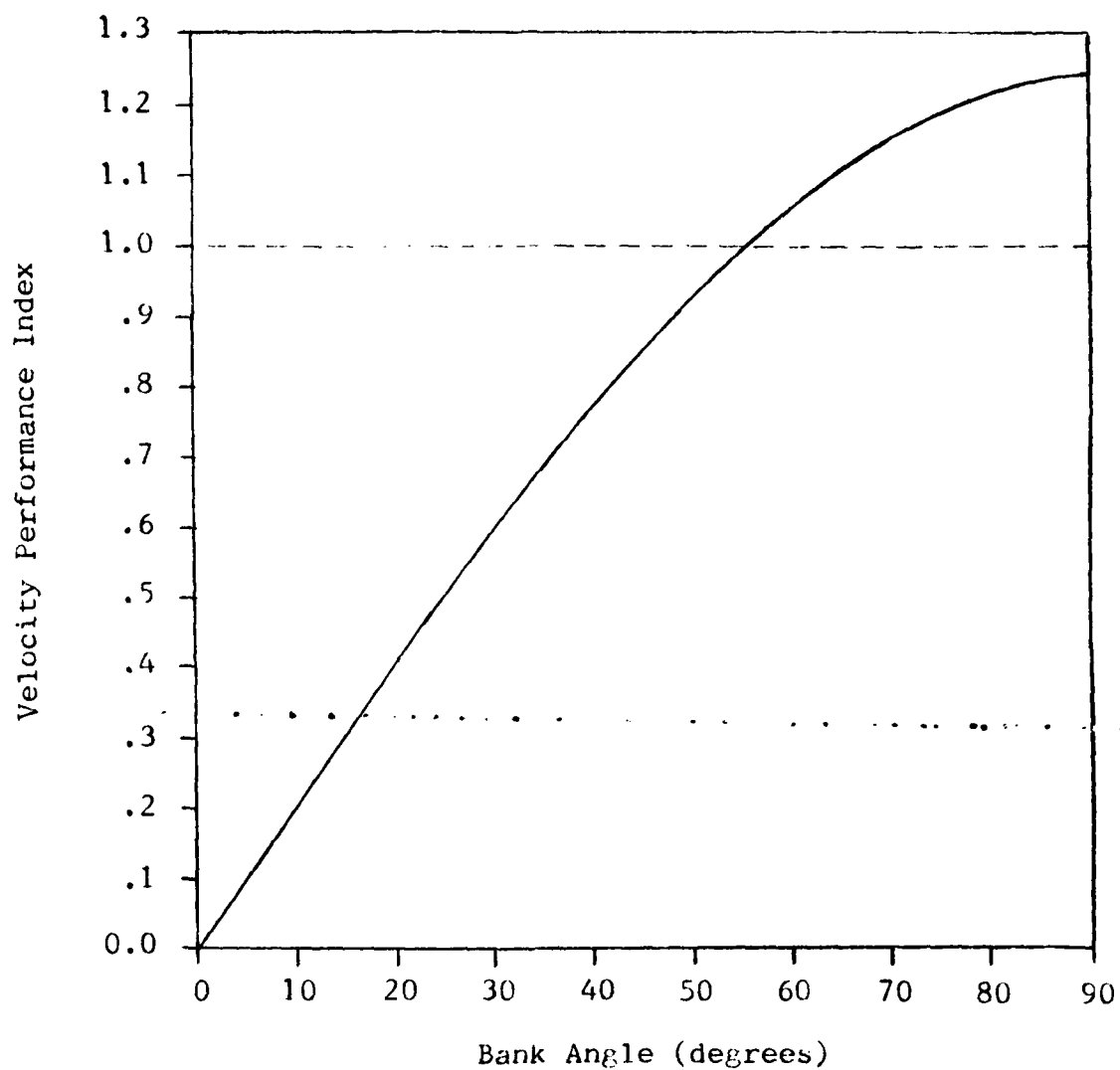


Figure V.5. Velocity Performance Index for 80 km Perigee Altitude

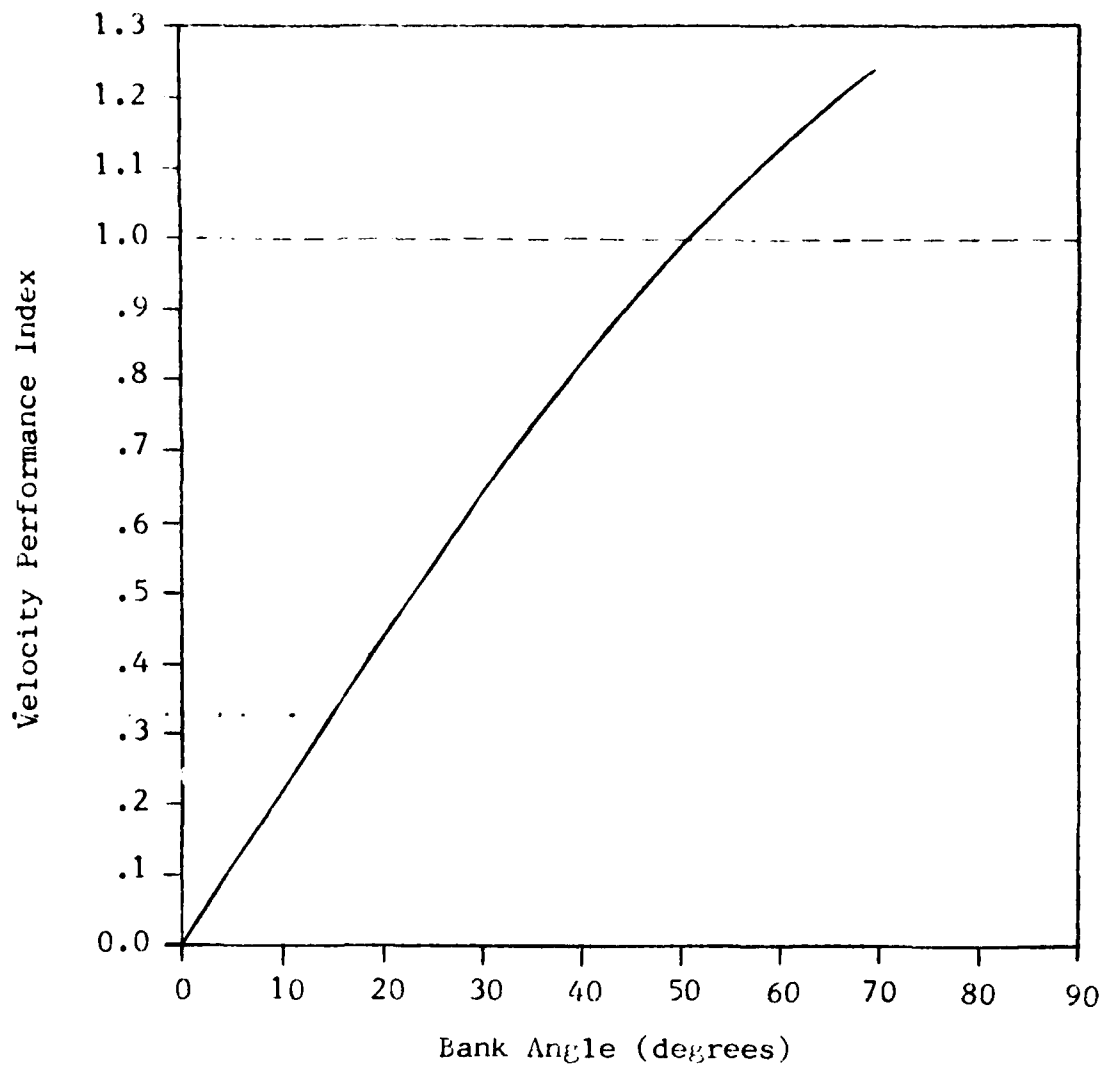


Figure V.6. Velocity Performance Index for 75 km Perigee Altitude

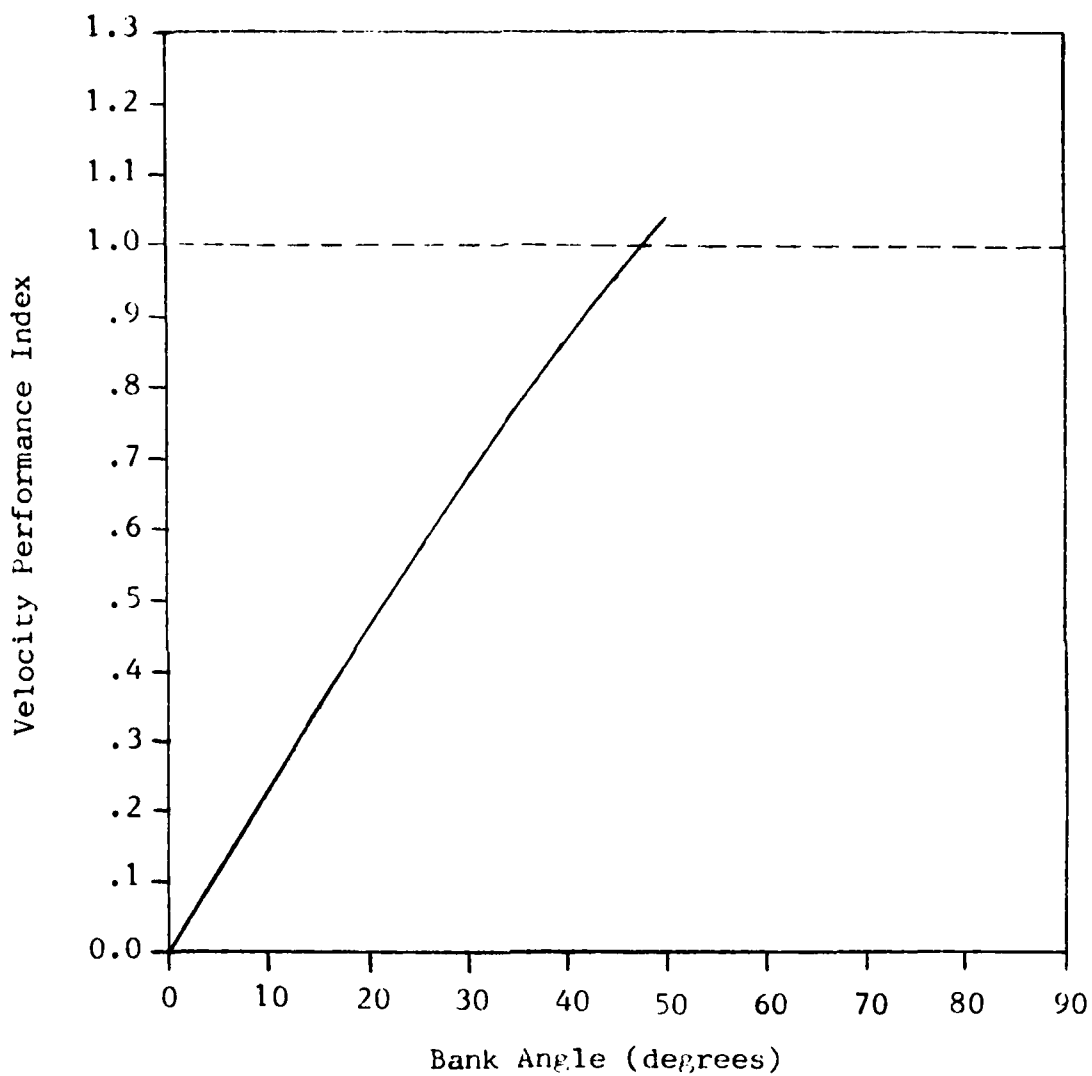


Figure V.7. Velocity Performance Index for 70 km Perigee Altitude

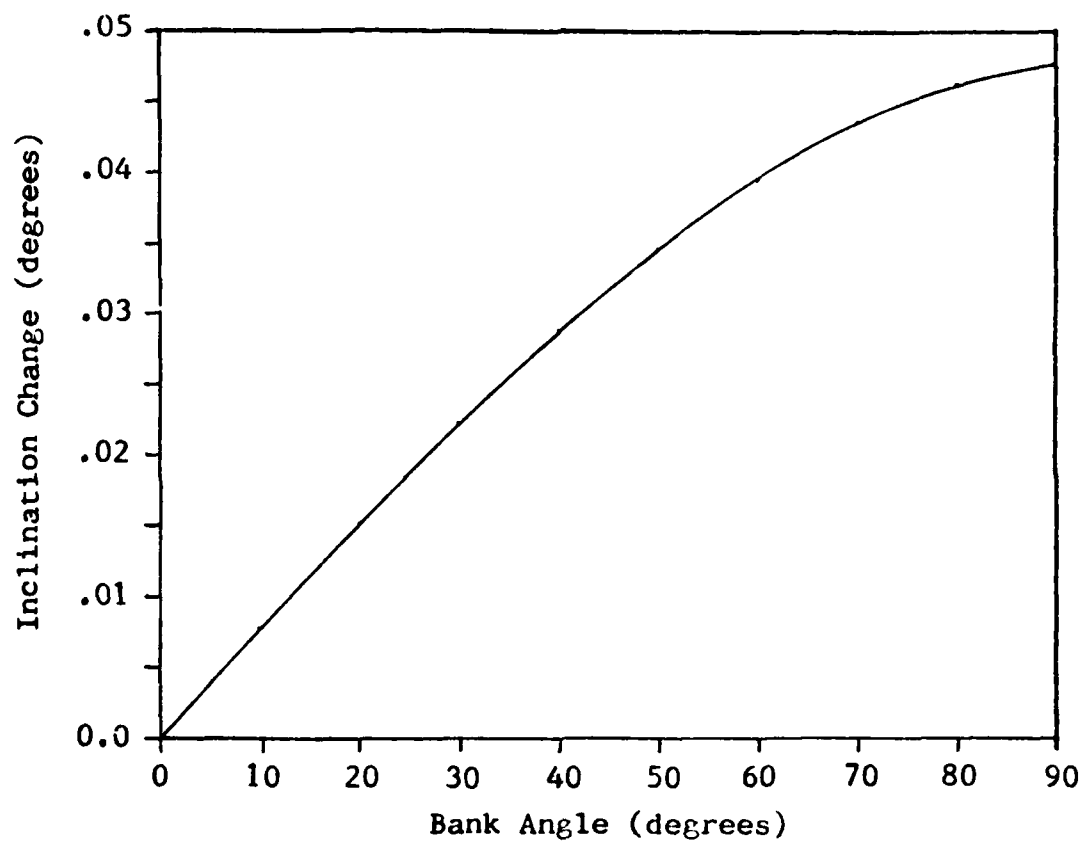


Figure V.8. Inclination Change for 95 km Perigee Altitude

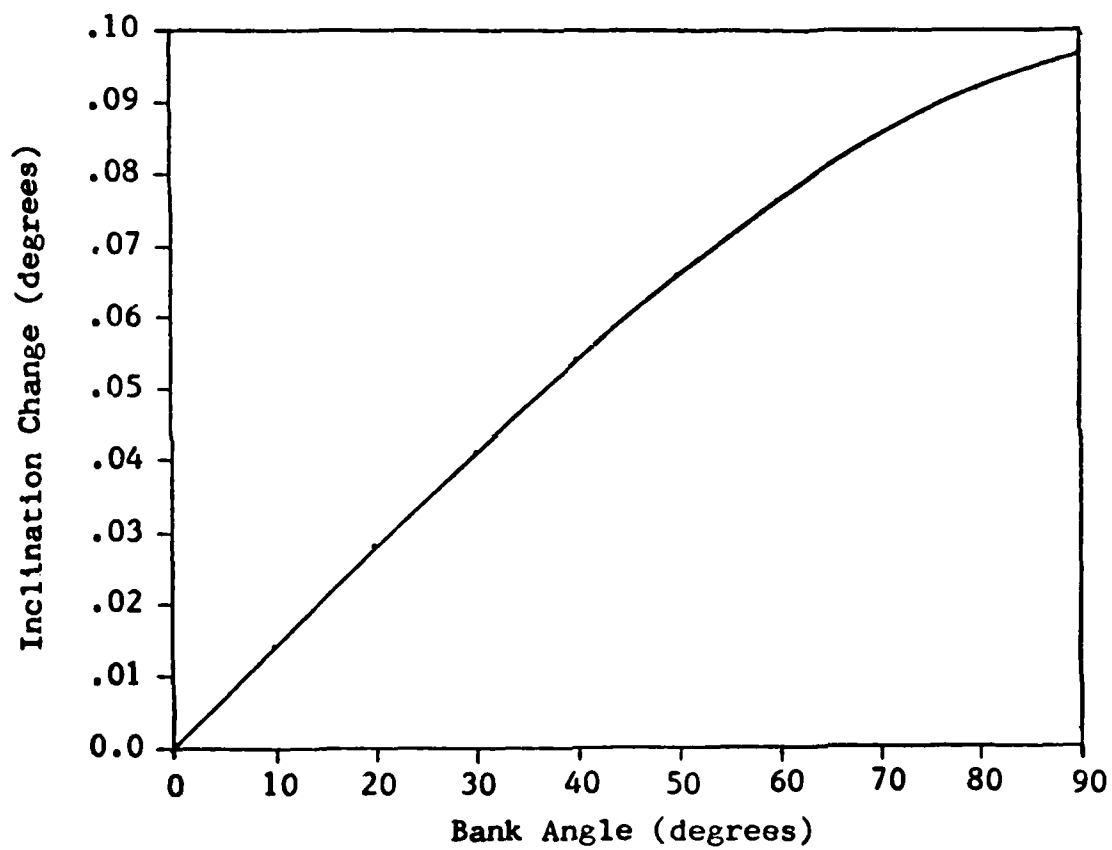


Figure V.9. Inclination Change for 90 km Perigee Altitude

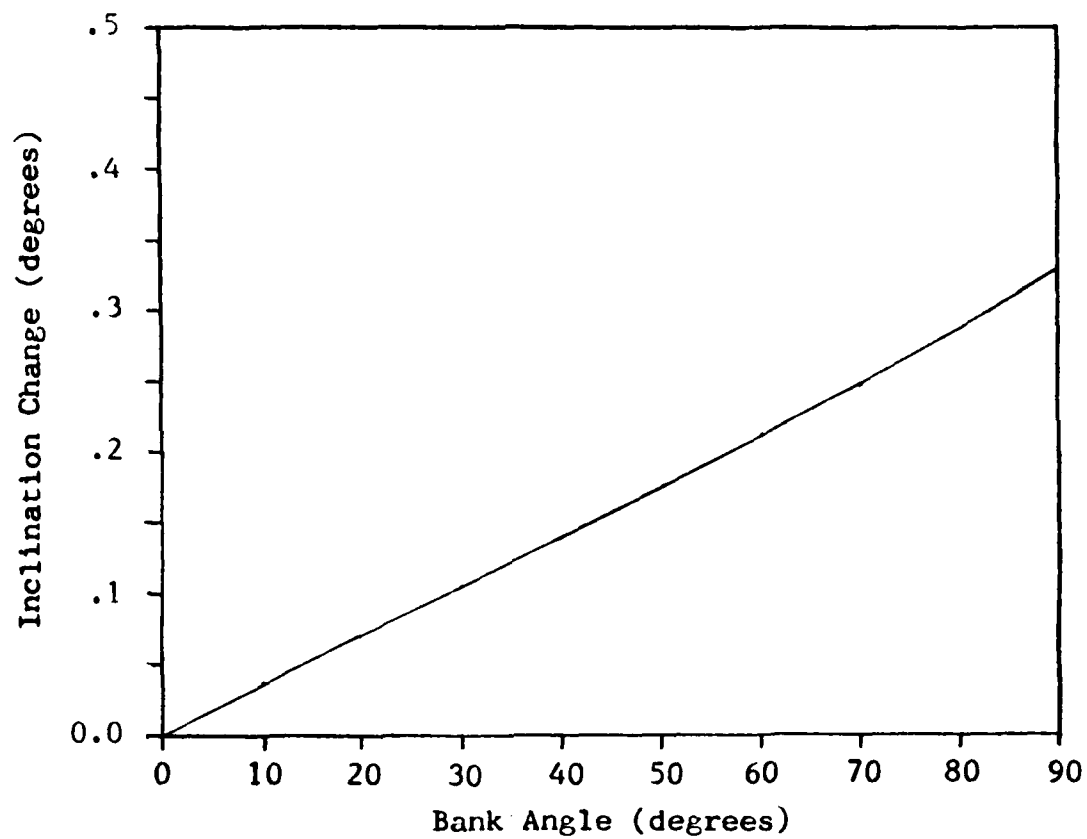


Figure V.10. Inclination Change for 85 km Perigee Altitude

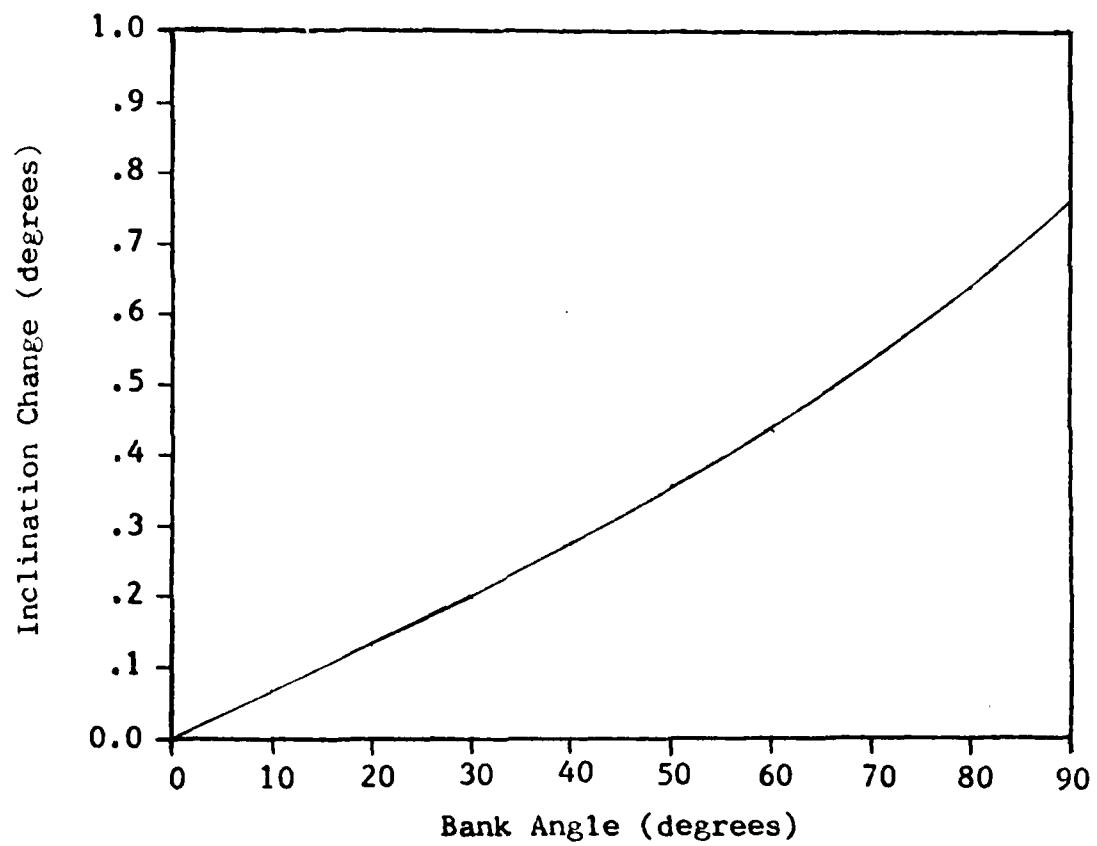


Figure V.11. Inclination Change for 80 km Perigee Altitude

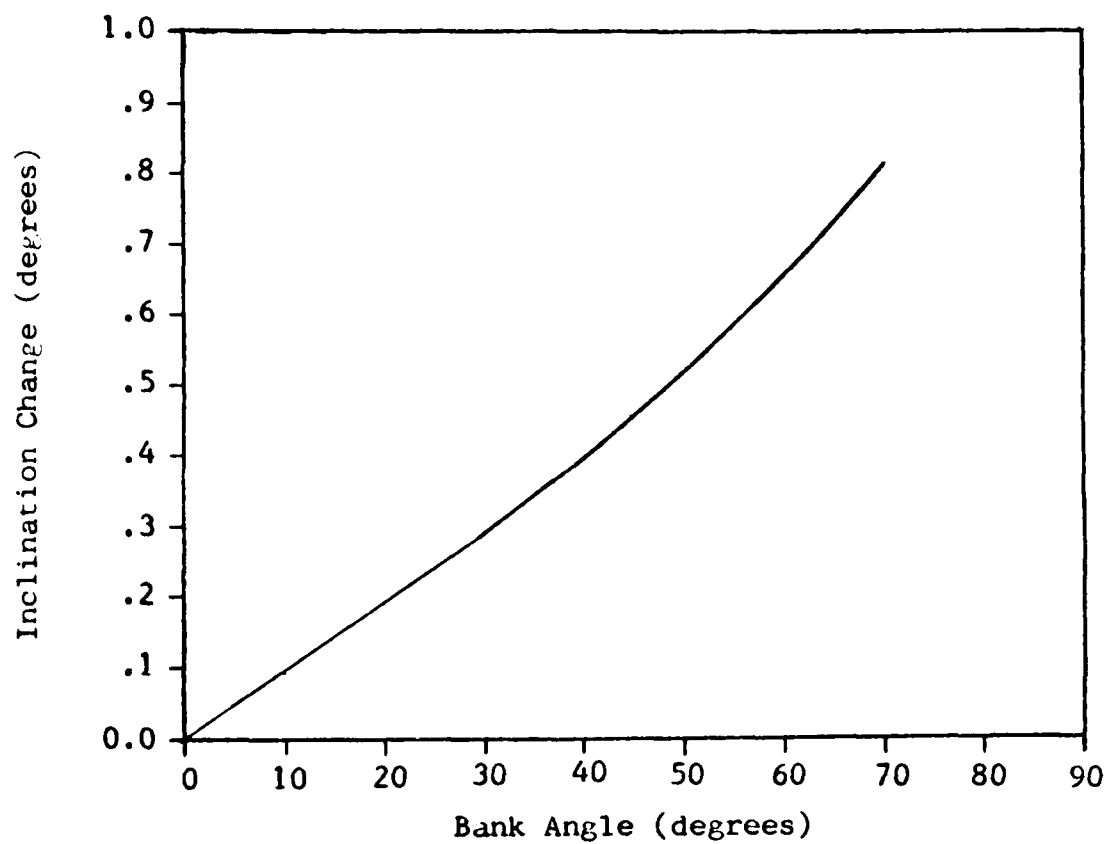


Figure V.12. Inclination Change for 75 km Perigee Altitude

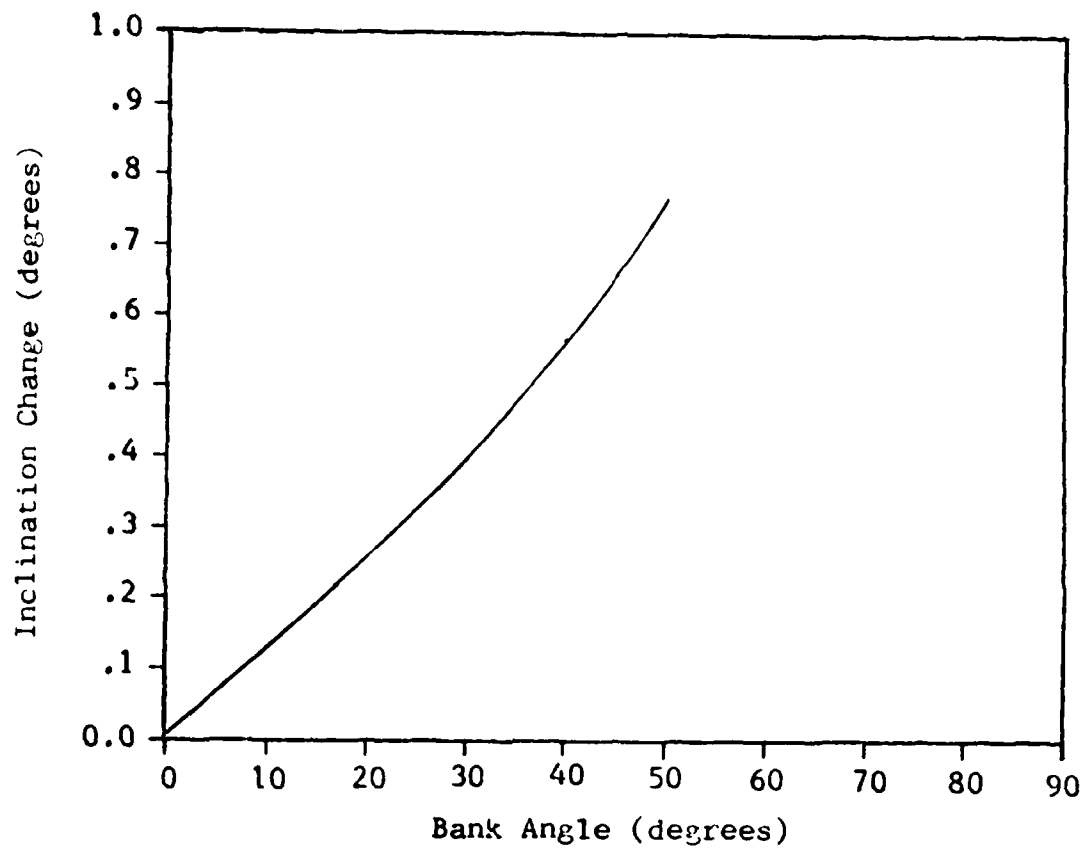


Figure V.13. Inclination Change for 70 km Perigee Altitude

## VI Optimal Control Technique

### Background

Obtaining a change in orbit inclination either an aerodynamic turn or a rocket burn in space results in a penalty. In both types of maneuvers the penalty is energy that must be supplied to the system in order to obtain the desired inclination change without changing the other orbit parameters. The most efficient method to change an orbit inclination outside the atmosphere is known: fire the rockets in a direction perpendicular to the orbit plane. On the other hand, the optimum angle of attack and bank angle control histories for an aerodynamic turn to effect a change in orbit inclination are not easy to determine and are presently unknown.

An aerodynamic skip turn reentry maneuver results in energy lost due to air drag. In order to change only the orbit inclination, the energy lost to air drag must be made up. This is accomplished by accelerating the orbiter in the new orbit plane. Thus, an optimum aerodynamic maneuver requires that the energy lost to air drag be minimized for a specified change in orbit inclination.

### Suboptimal Control

Suboptimal control theory, also called second-order parameter-optimization, is a second-order method which assumes that optimal controls can be approximated by polynomial functions of time (Ref 5, 6). For this study the control variables are angle of attack and bank angle. The approximating polynomials

are the Chebyshev polynomials which are chosen because they are orthogonal on the nondimensional time interval, zero to one. The Chebyshev polynomials will insure that the matrices in the suboptimal control scheme do not become singular due to possible linear dependence of the polynomials.

In essence, the suboptimal control problem is to find the coefficients of the control polynomials, subject to the differential constraints (equations of motion) and the prescribed boundary conditions (initial state, final altitude and orbit plane change), which minimize the performance index.

#### List of Symbols

$B$  = vector of control polynomial coefficients

$G$  = scalar performance index

$E$  = vector of prescribed final conditions

$\lambda$  = vector of Lagrange multiplier constants

$\delta$  = variational derivative

This study uses the subscript notation to denote partial derivatives, i.e.  $G_B$  indicates the partial derivatives of the scalar performance index with respect to the polynomial coefficients.

#### Suboptimal Control Formulation and Procedure

An augmented performance index,  $F$ , can be formulated by combining the performance index  $G$  with the equality constraints  $E$ .

$$F(B, \lambda) = G(B) + \lambda E(B)$$

The Lagrange multipliers,  $\lambda$ , are introduced to insure that linear dependence does not occur. The conditions to be satisfied at an extremal point are:

$$F_B^T(B, \lambda) = 0$$

$$E(B) = 0$$

The numerical procedure starts with guessed values for  $B$  and  $\lambda$ .  $F_B$  and  $E$  are linearized about these guessed values.

$$\delta F_B^T = F_{BB} \delta B + E_B^T \delta \lambda$$

$$\delta E = E_B \delta B$$

These expressions yield  $\delta B$  and  $\delta \lambda$  which result in  $F_B^T$  and  $E$  moving toward zero. Further, since  $\delta(\ ) = (\ )_{\text{new}} - (\ )_{\text{old}}$  and it is desired that  $(\ )_{\text{new}} = 0$ , then

$$\delta F_B^T = -P F_B^T$$

$$\delta E = -Q E$$

where  $P$  and  $Q$  are scalar factors which control the convergence process.  $P$  and  $Q$  take on values from near zero to one.

Combining the previous equations and solving for  $\delta B$  and  $\delta \lambda$  yields:

$$\delta \lambda = (E_B F_{BB}^{-1} E_B^T)^{-1} (-P E_B F_{BB}^{-1} F_B^T + Q E)$$

$$\delta B = -F_{BB}^{-1} (P F_B^T + E_B^T \delta \lambda)$$

where

$$F_B = G_B + \lambda E_B$$

$$F_{BB} = G_{BB} + \lambda E_{BB}$$

These partial derivatives are evaluated numerically using central differencing techniques.

In summary, the algorithm for the second-order suboptimal control scheme is as follows:

1. Guess  $B$  and  $\lambda$
2. Integrate the system equations of motion from an initial state to a final state
3. Compute  $E$ ,  $E_B$ ,  $F_B$ ,  $F_{BB}$
4. Select  $P$  and  $Q$
5. If  $F_B = 0$ ,  $F_{BB}$  is positive definite, and  $\delta B$  is small, the method has converged
6. If convergence criteria have not been satisfied, set  $B = B + \delta B$  and  $\lambda = \lambda + \delta \lambda$  and iterate procedures 2. through 6.

The procedure is straightforward, but some discussion is necessary concerning initial guesses for  $\lambda$  and  $B$  and selecting  $P$  and  $Q$ .

A good initial guess for  $\lambda$  can be obtained from a gradient approach. The equations for the gradient method are obtained by setting  $F_{BB} = I$  and  $\delta \lambda = 0$ . By manipulating the previous equations, an expression for  $\lambda$  can be found:

$$\lambda = (E_B E_B^T)^{-1} (Q/P)E - E_B G_B^T$$

The iteration procedure can be started with small values for  $P$  and  $Q$  (approximately .1). When it is apparent that the procedure is converging,  $P$  and  $Q$  can be increased. As soon as

the end conditions are satisfied,  $Q$  should be set equal to one in order to preserve the final conditions.

The guess at the coefficients contained in the  $B$  vector must be close to the actual optimal solution. The method used in this study is to initially consider the case for zero plane change with a constant angle of attack. Thus only one coefficient and one control polynomial need be considered. The optimal constant angle of attack is a good initial guess for a linear angle of attack polynomial. When the process converges on the optimum linear angle of attack polynomial, this resulting polynomial is used as an initial guess for a quadratic control polynomial, and so on until the next higher order polynomial does not substantially change the control history. The next step involves specifying a small change in orbit inclination and making a guess at a constant bank angle history while using the previous optimal angle of attack polynomials. Once the process converges for this new  $B$ , the suboptimal control method is continued with a constant bank angle history while specifying successively larger changes in orbit inclination until the end conditions cannot be met. Using this  $B$  as an initial guess for the largest orbit inclination change, higher order polynomials for the bank angle control can be found in a manner similar to that described previously for the angle of attack control polynomial.

## VII Conclusions and Recommendations

### Maximum L/D Analysis

The analysis and consequent results in Chapter V, though approximate, indicate that a skip turn reentry maneuver is profitable for all perigee altitudes between 70 and 95 km. The resulting changes in orbit inclination are approximately .8 degrees; however, larger changes can be achieved if the trajectory of the orbiter allows for more time spent in the atmosphere.

### Suboptimal Control Analysis

Most of the time spent on this study concerned the optimal control problem which, unfortunately, yielded virtually no results. The problem considered in this study was: Find the angle of attack and bank angle control histories that minimize the work done by air drag while meeting the specified end conditions, orbit plane change and final altitude. Though this problem is certainly relevant and important, it is poorly suited to suboptimal control. Three major problems occurred during the course of this study. First, the final time had to be specified. A good value for the time could be found in the case of zero plane change: the time the orbiter spent between entering and exiting the earth's atmosphere. However, once an orbit plane change was specified, a good value for the time was no longer available. Second, the final altitude had to be specified. This problem came into focus when it was

discovered that the suboptimal control scheme was causing the orbiter to lose energy just so the final altitude constraint could be met. The unsolved question here is: How is a final altitude established which is compatible with the desired orbit plane change?

Third, at the higher altitudes, 95 and 100 km, the change in the performance index with respect to the control polynomial coefficients was on the order of  $10^{-2}$  when the coefficients started approaching the optimal solution. This led to difficulties in evaluating the second-order partial derivatives which are necessary for the second-order suboptimal control scheme.

The problem can be restated so that these difficulties are alleviated. The restated problem follows: Find the angle of attack and bank angle time histories that maximize the change in orbit inclination (minimize  $-\Delta i$ ) for a given acceptable loss in energy. Instead of integrating the equations of motion to a fixed final time, integrate to a final altitude (equal to the initial altitude). Thus the loss in energy due to drag would be all kinetic energy. This will eliminate the first two difficulties encountered with the current problem. In order to alleviate the third difficulty it is recommended that the analysis be restricted to 90 km perigee altitude and lower. The probable difficulty with this newly-proposed problem will be establishing typical losses in energy that must be prescribed. This problem is not insurmountable. Typical values for energy loss can be found using an analysis similar to that formed in Chapter V of this study.

Due to the importance and interesting nature of a study such as this one, it is recommended that optimum skip turn reentry trajectories be investigated further. The newly-proposed problems stated previously are relevant and also warrant further and more accurate investigation.

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### Appendix A: Lockheed Bridging Formula

The Lockheed Bridging Formula is used to evaluate the aerodynamic force coefficients for the Space Shuttle Orbiter. The formula bridges the transitional flow regime from continuum flow to free-molecular flow. The formula is:

$$C_{trans} = C_{cont} + (C_{F.M.} - C_{cont}) \sin^n(\pi(A + B \log_{10} K_n))$$

where

$C_{cont}$  = viscous force coefficient values at  $VBAR = 0.08$

$C_{F.M.}$  = viscous force coefficient values at  $VBAR = 5.2$   
(free-molecular flow)

$$n = 2$$

$$A = 3/8$$

$$B = 1/8$$

$$K_n = \text{Knudsen number} = \lambda / L_{ref}$$

$$L_{ref} = 12.059 \text{ meters}$$

$$\lambda = \text{mean free path} = RT/P (2 N \sigma^2)^{-1}$$

$$R = \text{universal gas constant} = 8.314 \times 10^3 \text{ N-m/kg K}$$

$$T = \text{temperature}$$

$$P = \text{pressure}$$

$$N = \text{Avogadro's number} = 6.022 \times 10^{26} \text{ kmol}^{-1}$$

$$\sigma = \text{effective molecular collision diameter} \\ = 3.65 \times 10^{-10} \text{ meters}$$

## Appendix B: Bivariate Interpolation

The four point bivariate interpolation scheme is used to find the aerodynamic coefficients between the tabulated values. The interpolating formula is:

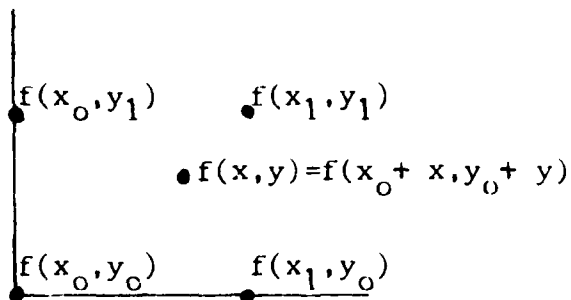
$$\begin{aligned} f(x,y) = f(x_0 + x, y_0 + y) = & (1 - p)(1 - q) f(x_0, y_0) \\ & + p(1 - q) f(x_1, y_0) + q(1 - p) f(x_0, y_1) \\ & + pq f(x_1, y_1) + \Theta(h^2) \end{aligned}$$

where

$$p = \delta x / \Delta x$$

$$q = \delta y / \Delta y$$

The four point interpolation scheme can be show graphically:



where

$$\Delta x = x_1 - x_0$$

$$\Delta y = y_1 - y_0$$

$$\delta x = x - x_0$$

$$\delta y = y - y_0$$

For this study,  $x$  and  $y$  correspond to VBAR and angle of attack respectively.

### Vita

Roger Robert Joseph Harding was born 4 August 1955 in Angola, Indiana. He graduated from Eaton Rapids High School in 1973 and attended the University of Michigan from which he received the degree of Bachelor of Science in Aerospace Engineering in April, 1977. Upon graduation, he received a commission in the United States Air Force through the ROTC program. He went on active duty with his entry in the School of Engineering, Air Force Institute of Technology, in September, 1977.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  Two types of skip reentry trajectories are examined in the 70 km to 95 km altitude region. The first is a maximum lift-to-drag analysis which indicates that an aerodynamic maneuver in order to change the orbit inclination is profitable when compared to a rocket burn in space to effect the same change in orbit inclination. The maximum changes in orbit inclination achieved aerodynamically were approximately .8 degrees. The second type of analysis considered the optimal control problem for a skip reentry trajectory. The		

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A specific problem posed was: Find the optimum angle of attack and bank angle controls which minimize the amount of work done by drag for a specific change in orbit inclination. No results were obtained from this analysis due to the problems encountered when the optimization technique was applied to the specific problem.

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